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Mass and Total Decay Rate of Unstable Particles in Quantum Field Theory†

MICHAEL RAM

Department of Physics, State University of New York, Buffalo, New York

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Abstract

We derive a very simple expression for the total decay rate of an unstable particle analogous to the optical theorem, and demonstrate its equivalence with the total decay rate defined in terms of the imaginary part of the propagator pole. The common origin of mass shifting and total decay rate is also demonstrated.

1. *Introduction*

In this paper, we would like to derive a very simple relation for the *total* decay rate of an unstable particle analogous to the optical theorem. We will show that this relation is identical with the *total* decay rate defined in terms of the imaginary part of the propagator pole to second order in the coupling constant, in the case of the Lee model with unstable V-particle and for unstable spin 0 and spin 1/2 particles with arbitrary interactions. The common origin of mass shifting and total decay rate will also be demonstrated. We work within the context of quantum field theory and use Feynman diagram methods. Natural units ($\hbar = c = 1$) are used throughout the paper

2. *Definition of Total Decay Rate of Unstable Particle in Terms of Time-Rate Change of Decay Probability*

Consider a system described by a unitary S-matrix S. Define the T-matrix as follows:

$$S = 1 + iT \tag{2.1}$$

Unitarity of the S-matrix implies that

$$T^* T = i(T^* - T) \tag{2.2}$$

† Work supported in part by grants from the Research Foundation of the State of New York and the National Science Foundation.

where * denotes hermitian conjugation. Let $|\mathbf{p};a\rangle$ be an unstable single particle state.† The *total* decay probability of this state in time τ is

$$P(E;a) = 1 - |\langle\mathbf{p};a|\,S\,|\mathbf{p};a\rangle|^2 \tag{2.3}$$

The second term on the right-hand side of this relation is simply the probability that the particle has not decayed. To see how the time τ occurs in equation (2.3), one should think of S as replaced by the time development operator $U(\tau/2, -\tau/2)$ with the limit $\tau \to \infty$ implied at the end of the calculation. This procedure is quite common in the Dyson–Feynman treatment of perturbation theory which we are invoking (Schweber, 1961). We would also like to make the following remarks concerning the definition of the state $|\mathbf{p};a\rangle$. Since, as stated in Section 1, we work in the context of perturbation theory and use Feynman diagram techniques, this method requires us in general to invoke the adiabatic hypothesis. This hypothesis in turn implies that the state $|\mathbf{p};a\rangle$ is simply an eigenstate of the unperturbed Hamiltonian. One can justifiably ask whether invoking the adiabatic hypothesis is appropriate in the description of an unstable particle state which is not a stationary state. Our answer to this is in the affirmative, for the following reasons. If we restrict ourselves to lowest-order perturbation theory and calculate $P(E;a)$ to second order in the coupling constant, then to this order $|\mathbf{p};a\rangle$ is precisely the unperturbed state. As shown by Lévy (1959), the total decay rate of an unstable particle is only well defined to second order in the coupling constant, and there is an intrinsic uncertainty which is of the fourth order. Because of this, we shall limit our calculations to second order in the coupling constant. To this order, the use of the unperturbed state for $|\mathbf{p};a\rangle$ is legitimate, and in fact exact.‡ Substituting equation (2.1) into relation (2.3) we find that

$$\begin{aligned} P(E;a) &= 2\,\mathrm{Im}\,\langle\mathbf{p};a|\,T\,|\mathbf{p};a\rangle - |\langle\mathbf{p};a|\,T\,|\mathbf{p};a\rangle|^2 \\ &= -2\,\mathrm{Re}\,\langle\mathbf{p};a|\,S-1\,|\mathbf{p};a\rangle - |\langle\mathbf{p};a|S-1\,|\mathbf{p};a\rangle|^2 \end{aligned} \tag{2.4}$$

Relation (2.4) can also be derived in a slightly different way starting from

$$\begin{aligned} P(E;a) &= \sum_{|f\rangle \neq |p;a\rangle} |\langle f|\,T\,|\mathbf{p};a\rangle|^2 \\ &= \sum_{|f\rangle} \langle\mathbf{p};a|\,T^*|f\rangle\langle f|\,T\,|\mathbf{p};a\rangle - |\langle\mathbf{p};a|\,T\,|\mathbf{p};a\rangle|^2 \end{aligned} \tag{2.5}$$

The summation is over a complete set of states $|f\rangle$. Using the completeness relation $|f\rangle\langle f| = 1$, together with equation (2.2), one easily checks that equations (2.4) and (2.5) are identical.

Since we are working to second order in the coupling constant we can drop the term $|\langle\mathbf{p};a|T|\mathbf{p};a\rangle|^2$ in equation (2.4), since the lowest-order

† $p = (\mathbf{p}, iE)$ is the 4-momentum of the state and a stands for any other quantum numbers required for its identification (a may simply be, for example, the given name of the particle or resonance).

‡ The use of the adiabatic hypothesis in calculating decay rates to second order in the coupling constant is quite common in the literature.

contribution to it is the fourth.† To second order in the coupling constant we can therefore write

$$P(E;a) = 2\,\mathrm{Im}\langle \mathbf{p};a|T|\mathbf{p};a\rangle \qquad (2.6)$$
$$= -2\,\mathrm{Re}\langle \mathbf{p};a|S-1|\mathbf{p};a\rangle$$

We now define the total decay rate of the state $|\mathbf{p};a\rangle$ as ‡

$$\Gamma(E;a) = \frac{1}{\tau}P(E;a) \qquad (2.7)$$

Combining equations (2.6) and (2.7) we find that

$$\Gamma(E;a) = \frac{2}{\tau}\mathrm{Im}\langle \mathbf{p};a|T|\mathbf{p};a\rangle$$
$$= -\frac{2}{\tau}\mathrm{Re}\langle \mathbf{p};a|S-1|\mathbf{p};a\rangle \qquad (2.8)$$

This result is correct to second order in the coupling constant.

Equation (2.8) shows that the total decay rate is completely determined by $\mathrm{Im}\langle \mathbf{p};a|T|\mathbf{p};a\rangle$. This is exactly analogous to the optical theorem, which relates the *total* cross-section for scattering of two particles a and b in the state $|\mathbf{p}_a,a;\mathbf{p}_b,b\rangle$ to $\mathrm{Im}\langle \mathbf{p}_a,a;\mathbf{p}_b,b|T|\mathbf{p}_a,a;\mathbf{p}_b,b\rangle$. [In a similar way, it is quite easy to see that the total reaction (or interaction) probability for scattering of N particles is simply related to $\mathrm{Im}\langle \mathbf{p}_1,a_1,\ldots,\mathbf{p}_N,a_N|T|\mathbf{p}_1,a_1,\ldots,\mathbf{p}_N,a_N\rangle$.]

It would seem that the definition (2.7), and therefore also relation (2.8), is only well suited for unstable particles whose decay rates can be measured directly by observing the number of particles that have decayed after a certain time has elapsed. We shall later in the paper present arguments which make it feasible to adopt relation (2.8) also for hadron resonances which do not satisfy such a criterion.

Relation (2.8) is extremely interesting, since it represents a *total* decay rate and therefore includes both dynamic and kinematic effects. Another very attractive feature is its simplicity. In the next section we will show that relation (2.8) coincides with the total decay rate defined in terms of the imaginary part of the propagator pole to second order in the coupling constant. We will demonstrate this in the case of the Lee model with unstable V-particle, and for unstable spin 0 and spin 1/2 particles with arbitrary interactions.

3. *Applications*

A. *The Lee Model with Unstable V-Particle*

The Lee model with unstable V-particle (Glaser & Källen, 1957) is particularly well suited for our purpose, since V can only decay into N and

† This will be quite amply demonstrated in the applications of Section 3.

‡ When one calculates $P(E;a)$ to second order in the coupling constant using the Dyson–Feynman perturbation method one finds that it is linear in τ (Schweber, 1961). This will be demonstrated in the applications of Section 3. As a result, $\Gamma(E;a)$ defined by relation (2.7) is independent of τ.

θ so that partial and total decay rates are identical. In this way, it is quite easy to test relation (2.8) directly by calculating the decay rate of $V \to N\theta$, and comparing the result with the right-hand side of relation (2.8). This will be done using conventional Feynman diagram techniques, and carried out to second order in the coupling constant.

The Lee model Hamiltonian is (Glaser & Källen, 1957)

$$H = H_0 + H_I \tag{3.1}$$

where

$$H_0 = m_V \int \psi_V^*(x)\psi_V(x)\,d\mathbf{x} + m_N \int \psi_N^*(x)\psi_N(x)\,d\mathbf{x} + \tfrac{1}{2}\int [\pi^2(x) + \{\nabla\phi(x)\}^2 + \mu^2\phi^2(x)]\,d\mathbf{x} \tag{3.2a}$$

and

$$H_I = g\int [\psi_V^*(x)\psi_N(x)A(x) + \psi_N^*(x)\psi_V(x)A^*(x)]\,d\mathbf{x} + \delta m_V \int \psi_V^*(x)\psi_V(x)\,d\mathbf{x} \tag{3.2b}$$

The fields $\psi_V(x)$ and $\psi_N(x)$ are those associated with the V and N particles respectively, and satisfy the usual equal time anticommutation relations. Note that we use the 4-vector notation $x = (\mathbf{x}, it)$. The field $\phi(x)$ is the scalar field associated with θ particles, and $\pi(x)$ its canonically conjugate momentum density. We use the notation m_V, m_N and μ for the 'physical' masses of the V, N and θ particles respectively, and δm_V for the V-particle mass renormalization counter term. Also†

$$A(x) = \sum_{\mathbf{k}} \frac{f(\omega)}{\sqrt{(2\omega\Omega)}}\alpha_{\mathbf{k}}\exp(ik.x) \tag{3.3}$$

where the $\alpha_{\mathbf{k}}$ are defined through

$$\phi(x) = \sum_{\mathbf{k}} \frac{1}{\sqrt{(2\omega\Omega)}}\{\alpha_{\mathbf{k}}\exp(ik.x) + \alpha_{\mathbf{k}}^*\exp(-ik.x)\} \tag{3.4}$$

and

$$k.x = \mathbf{k}.\mathbf{x} - \omega t; \qquad \omega = (\mathbf{k}^2 + \mu^2)^{1/2}$$

The function $f(\omega)$ is a cutoff function which is introduced in order to make all quantities finite and well defined. The coupling constant for the reaction $V \rightleftarrows N\theta$ is g. We assume that $m_V > m_N + \mu$ so that the V-particle can decay spontaneously into N and θ.

Consider the decay of a V-particle of 4-momentum $p = (\mathbf{p}, ip_0)$ into an N and θ particle of 4-momentum $p' = (\mathbf{p}', ip_0')$ and $k = (\mathbf{k}, i\omega)$ respectively. The lowest-order Feynman diagram for this process is given in Fig. 1(a).

† The plane wave functions we use are normalized in a box of volume Ω.

Applying the well-known Feynman techniques, the S-matrix element for this process is

$$S^{(1)}(V \to N\theta; p_0 = m_V) = -\frac{i(2\pi)^4 gf(\omega)}{\Omega\sqrt{(2\omega\Omega)}}\delta^{(4)}(p' + k - p) \qquad (3.5)$$

The *total* decay probability in time τ is

$$\begin{aligned} P(p_0 = m_V; V) &= \sum_{p'}\sum_{k} |S^{(1)}(V \to N\theta; p_0 = m_V)|^2 \\ &= \frac{g^2}{2\pi}\{f(m_V - m_N)\}^2\sqrt{[(m_V - m_N)^2 - \mu^2]}\,\tau \end{aligned} \qquad (3.6)$$

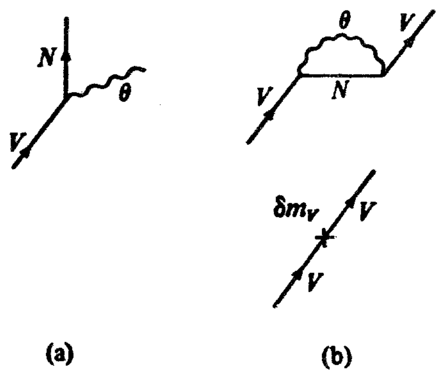

Figure 1—(a) Lowest order Feynman diagram for decay of V particle, (b) V-particle self-energy effect and corresponding mass counterterm effect.

and the *total* decay rate

$$\Gamma(p_0 = m_V; V) = \frac{1}{\tau}P(p_0 = m_V; V) = \frac{g^2}{2\pi}\{f(m_V - m_N)\}^2\sqrt{[(m_V - m_N)^2 - \mu^2]} \qquad (3.7)$$

Let us now evaluate $\Gamma(p_0 = m_V; V)$ using relation (2.8). The only contribution of order g^2 to $\langle p_0 = m_V; V|S - 1|p_0 = m_V; V\rangle$ comes from the Feynman diagrams of Fig. 1(b) and is given by

$$\begin{aligned} S^{(2)}(V \to V, p_0 = m_V) &= \langle p_0 = m_V; V|S - 1|p_0 = m_V; V\rangle \\ &= -i\tau\{\textstyle\sum(V; p_0 = m_V) + \delta m_V\} \end{aligned} \qquad (3.8)$$

where

$$\begin{aligned} \textstyle\sum(V; p_0) &= g^2\sum_{k}\frac{\{f(\omega)\}^2}{2\omega\Omega}\frac{1}{(p_0 - m_N - \omega + i\epsilon)} \\ &= \textstyle\sum_R(V; p_0) + i\sum_I(V; p_0) \end{aligned} \qquad (3.9)$$

In the above, ϵ is a positive, infinitesimal real number, and $\sum_R (V;p_0)$ and $\sum_I (V;p_0)$ are both real and given by

$$\sum_R (V;p_0) = -\frac{g^2}{(2\pi)^2} P \int\limits_{\mu}^{\infty} \frac{\sqrt{(\omega^2-\mu^2)}\{f(\omega)\}^2}{\{\omega-(p_0-m_N)\}} d\omega \tag{3.10a}$$

$$\sum_I (V;p_0) = -\frac{g^2}{4\pi}\{f(p_0-m_N)\}^2\sqrt{[(p_0-m_N)^2-\mu^2]} \tag{3.10b}$$

P in equation (3.10a) denotes the principle value. Substituting equations (3.8) and (3.9) into the right-hand side of relation (2.8) and using equation (3.10b), we find that

$$\Gamma(p_0=m_V;V) = -2\sum_I (V;p_0=m_V) = \frac{g^2}{(2\pi)}\{f(m_V-m_N)\}^2 \times \sqrt{[(m_V-m_N)^2-\mu^2]} \tag{3.11}$$

which is identical with the result (3.7).

As a generalization of the mass renormalization condition common for stable particles (Ram & Rosen, 1963; Schweber, 1961), it is very natural to choose the corresponding condition here as

$$\operatorname{Im} S^{(2)}(V \to V;p_0=m_V)=0 \tag{3.12}$$

This implies that δm_V has to be chosen such that

$$\sum_R (V;p_0=m_V)+\delta m_V=0 \tag{3.13}$$

From relations (3.11) and (3.13), we see that

$$\sum (V;p_0=m_V) = -\left\{\delta m_V + i\frac{\Gamma(V)}{2}\right\} \tag{3.14}$$

clearly demonstrating the 'common' origin of mass shifting and total decay rate. This idea is not really new and has been well known for a very long time (Heitler, 1954).

In the absence of interaction, the V-particle propagator in momentum space is simply

$$\frac{i}{(2\pi)^4}S^{(0)}(V;p_0) = \frac{i}{(2\pi)^4}\frac{1}{(p_0-m_V^{(0)}+i\epsilon)} \tag{3.15}$$

where $m_V^{(0)}$ is the bare V-particle mass. In the presence of interaction, the V-particle propagator will be modified. One can readily show that to second order in the coupling constant, the modified propagator is

$$\frac{i}{(2\pi)^4}S'(V;p_0) = \frac{i}{(2\pi)^4}\frac{1}{(p_0-m_V)-\{\sum (V;p_0)+\delta m_V\}} \tag{3.16}$$

Using the mass renormalization condition (3.13), and keeping terms to order g^2 only, it is easy to show that in the vicinity of $p_0 = m_V$

$$S_V'(p_0) \approx \frac{Z_V}{(p_0 - m_V) + i\dfrac{\Gamma(p_0 = m_V; V)}{2}} \tag{3.17}$$

where

$$Z_V^{-1} = \frac{d}{dp_0}[S'(V;p_0)]^{-1}\bigg|_{p_0=m_V} = 1 - \frac{d\sum(V;p_0)}{dp_0}\bigg|_{p_0=m_V} \tag{3.18}$$

and $\Gamma(p_0 = m_V; V)$ is simply given by equation (3.11) and represents the total decay rate of the V-particle as calculated from relation (2.8) to second order in the coupling constant. If we calculate the $N\theta$ scattering cross-section it will be proportional to

$$|S'(V;p_0)|^2 \approx \frac{|Z_V|^2}{(p_0 - m_V)^2 + \left\{\dfrac{\Gamma(p_0 = m_V; V)}{2}\right\}^2} \tag{3.19}$$

for p_0 close to m_V. The cross-section will therefore exhibit a resonance at $p_0 = m_V$ with half width $\Gamma(p_0 = m_V; V)$, which is precisely the total decay rate of the V-particle as calculated from relation (2.8).

B. *Unstable Spin* 0 *Particle with Arbitrary Interaction*

Consider an unstable spin 0 particle π (not to be associated necessarily with the physical pion) of physical mass m_π and arbitrary interaction. Applying relation (2.8), the total decay rate of a π-particle of 4-momentum $k = (\mathbf{k}, i\omega)$ is

$$\Gamma(\omega; \pi) = -\frac{2}{\tau}\mathrm{Re}\langle \mathbf{k};\pi|S-1|\,\mathbf{k};\pi\rangle \tag{3.20}$$

To second order in the interaction, the only contribution to

$$\langle \mathbf{k};\pi|S-1|\,\mathbf{k};\pi\rangle$$

comes from the Feynman diagrams of Fig. 2, and is given by†

$$\langle \mathbf{k};\pi|S-1|\,\mathbf{k};\pi\rangle = \frac{i}{2\omega}\tau\{\textstyle\sum(k^2 = -m_\pi^2;\pi) + \Delta m_\pi^2\} \tag{3.21}$$

† Since we work to second order in the coupling constant the state $|\mathbf{k};\pi\rangle$ is to be taken as the unperturbed state which is an eigenstate of the energy with eigenvalue $\omega = (\mathbf{k}^2 + m_\pi^2)^{\frac{1}{2}}$.

where $\sum (k^2;\pi)$ represents the contribution of the self-energy 'blob' and Δm_π^2 is the (mass)^2 renormalization counterterm for the π. Substituting equation (3.21) into relation (3.20), we find that

$$\Gamma(\omega;\pi) = \frac{1}{\omega} \textstyle\sum_I (k^2 = -m_\pi^2;\pi) \tag{3.22}$$

where

$$\textstyle\sum (k^2;\pi) = \sum_R (k^2;\pi) + i \sum_I (k^2;\pi) \tag{3.23}$$

and $\sum_R (k^2;\pi)$ and $\sum_I (k^2;\pi)$ are both real. Imposing as mass renormalization condition (Ram & Rosen, 1963; Schweber, 1961)

$$\mathrm{Im}\, \langle \mathbf{k};\pi | S - 1 | \mathbf{k};\pi \rangle = 0 \tag{3.24}$$

requires that we choose

$$\Delta m_\pi^2 = -\textstyle\sum_R (k^2 = -m_\pi^2;\pi) \tag{3.25}$$

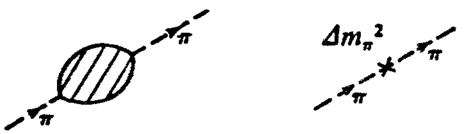

Figure 2—Second-order self-energy effect for π-particle and corresponding counterterm effect.

Combining relations (3.22), (3.23) and (3.25) gives

$$\textstyle\sum (k^2 = -m_\pi^2;\pi) = -\{\Delta m_\pi^2 - i\omega\Gamma(\omega;\pi)\} \tag{3.26}$$

clearly demonstrating the common origin of mass shifting and total decay rate.

We will now show that to second order in the coupling constant, the total decay rate as given by (3.22) is identical with the total decay rate defined in terms of the imaginary part of the propagator pole.

To second order in the coupling constant, the modified π-propagator is

$$\frac{-i}{(2\pi)^4} S'(p^2;\pi) = \frac{-i}{(2\pi)^4} \frac{1}{(p^2 + m_\pi^2) - \{\sum (p^2;\pi) + \Delta m_\pi^2\}} \tag{3.27}$$

where $p = (\mathbf{p}; iE)$. One can readily show that for E close to ω

$$S'(p^2;\pi) \approx \frac{-Z_\pi/2\omega}{(E-\omega) + i\left\{\dfrac{\Gamma(\omega;\pi)}{2}\right\}} \tag{3.28}$$

where $\Gamma(\omega;\pi)$ is given by relation (3.22), and is precisely the total decay rate of the π-particle as calculated from relation (2.8). Also

$$Z_\pi^{-1} = \frac{d}{dp^2}[S'(p^2;\pi)]^{-1}\Bigg|_{p^2=-m_\pi^2} = 1 - \frac{d\sum(p^2;\pi)}{dp^2}\Bigg|_{p^2=-m_\pi^2} \tag{3.29}$$

In any reaction in which the π-particle is produced as an intermediate state, the cross-section for E close to ω will be proportional to

$$|S'(p^2;\pi)|^2 \approx \frac{|Z_\pi/2\omega|^2}{(E-\omega)^2 + \left\{\frac{\Gamma(\omega;\pi)}{2}\right\}^2} \tag{3.30}$$

and will consequently exhibit a resonance of half width $\Gamma(\omega;\pi)$ about $E=\omega$.

C. *Unstable Spin 1/2 Particle with Arbitrary Interaction*

Consider an unstable spin 1/2 particle n (not to be confused with the neutron) of physical mass m_n. We shall assume that the interaction

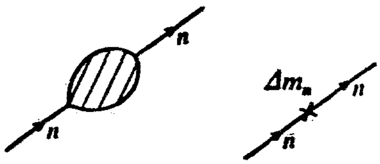

Figure 3—Second-order self-energy effect for n-particle and corresponding counterterm effect.

responsible for the decay of the n-particle is invariant under the full, inhomogeneous Lorentz group, but is otherwise arbitrary. This excludes, for example, the weak decays of spin 1/2 particles like the neutron which violate parity conservation. Applying relation (2.8), the total decay rate of an n-particle of 4-momentum $k=(\mathbf{k}, i\omega)$ is

$$\Gamma(\omega;n) = -\frac{2}{\tau}\mathrm{Re}\langle\mathbf{k};n|S-1|\mathbf{k};n\rangle \tag{3.31}$$

To second order in the interaction, the only contribution to

$$\langle\mathbf{k};n|S-1|\mathbf{k};n\rangle$$

comes from the Feynman diagrams of Fig. 3, and is given by

$$\langle\mathbf{k};n|S-1|\mathbf{k};n\rangle = -i\tau\bar{u}(k)\{\sum(k;n) - \Delta m_n\}u(k) \tag{3.32}$$

where $\sum(k;n)$ represents the contribution of the self-energy 'blob' and Δm_n is the mass counterterm for the n-particle. The spinors $u(k)$ and $\bar{u}(k) = u^*(k)\gamma_4$ satisfy the free Dirac equation in momentum space, i.e.,

$$(\not{k} - m_n)u(k) = 0; \qquad \bar{u}(k)(\not{k} - m_n) = 0 \tag{3.33}$$

where

$$\not{k} = -i\sum_{\mu=1}^{4}\gamma_\mu k_\mu$$

We use the noncovariant normalization

$$\bar{u}(k)u(k) = \frac{m_n}{\omega} \tag{3.34}$$

Because of the assumed invariance of the interaction under the full inhomogeneous Lorentz group, one can show that (Schweber, 1961)

$$\sum(k;n) = A + (\not{k} - m_n)B + (\not{k} - m_n)^2\sum^{(f)}(k) \tag{3.35}$$

where A and B are constants independent of k. Substituting equations (3.32) and (3.35) into relation (3.31) and using equations (3.33) and (3.34), we find that

$$\Gamma(\omega;n) = -2\frac{m_n}{\omega}A_I \tag{3.36}$$

where

$$A = A_R + iA_I \tag{3.37}$$

and A_R and A_I are both real.

Imposing as mass renormalization condition

$$\mathrm{Im}\langle \mathbf{p};n|S-1|\mathbf{p};n\rangle = 0 \tag{3.38}$$

we find that

$$\Delta m_n = A_R \tag{3.39}$$

Combining relations (3.36), (3.37) and (3.39) gives

$$A = \Delta m_n - i\frac{\omega}{2m_n}\Gamma(\omega;n) \tag{3.40}$$

which again points to the common origin of mass shifting and total decay rate.

We will now demonstrate that to second order in the coupling constant, the total decay rate as given by equation (3.36) is identical with the total decay rate defined in terms of the imaginary part of the propagator pole.

To second order in the coupling constant, the modified n-particle propagator is

$$\frac{i}{(2\pi)^4}S'(p;n) = \frac{i}{(2\pi)^4}\frac{1}{(\not{p} - m_n) - \{\sum(p;n) - \Delta m_n\}} \tag{3.41}$$

Using equations (3.35) and (3.39), one can readily show that in the vicinity of $E = \omega$

$$S'(p;n) \propto \frac{1}{(E-\omega) - \dfrac{2im_n A_I}{2\omega}} \tag{3.42}$$

where $\propto$ denotes proportionality. It is therefore clear that the cross-section for a reaction in which the n-particle appears as an intermediate state will be proportional to

$$\left[(E-\omega)^2 + \left\{\frac{\Gamma(\omega;n)}{2}\right\}^2\right]^{-1}$$

in the vicinity of $E = \omega$, and will therefore exhibit a resonance of half width $\Gamma(\omega;n)$ at $E = \omega$, where $\Gamma(\omega;n)$ is the total decay rate of the n-particle as calculated from relation (2.8).

4. *Discussion*

It would seem that the definition (2.7), and consequently also relation (2.8), is only well suited for unstable particles whose decay rates can be measured directly by observing the number of particles that have decayed after a certain time has elapsed. It is nevertheless feasible that relation (2.8) is more general than its derivation implies, and that it could be valid also for hadron resonances whose lifetimes are too short to be directly measured. Relations such as (3.14), (3.26) and (3.40) seem to give credence to such a possibility, since the common origin of mass shifting and total decay rate is generally accepted for all unstable particles, including hadron resonances. Since relation (2.8) includes both dynamic and kinematic effects, it is ideally suited for the important application of symmetry considerations. In fact, we have applied relation (2.8) to hadrons and derived (Ram, 1969) interesting relations among total decay rates of hadron resonances analogous to the Gell-Mann–Okubo formulae for masses.

We would finally like to point out that the equality of the total decay rate as given by relation (2.8) and the total decay rate defined in terms of the imaginary part of the propagator pole serves as a 'demonstration' of the energy-time uncertainty relation for the decay of unstable particles.

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